The Cross-Predictive Ability of Crowding: Does Beta Arbitrage Predict Momentum Profits?

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Abstract

This paper shows that high arbitrage activity in the betting-against-beta strategy (BAB) is a strong predictor of momentum profits. Following high levels of crowding in the BAB strategy, the momentum strategy yields -0.36% on average in the first month after portfolio formation. In contrast, following low levels of crowding in BAB, the momentum strategy yields 1.49% on average in the first month after portfolio formation. Going from low to high levels of BAB crowding, the odds of a momentum crash in the first month after portfolio formation increase 4.5 times. These results can be traced back to the overlap between winners and low-beta stocks and the overlap between losers and high-beta stocks. Furthermore, the positive-feedback trading mechanism embedded in BAB amplifies the crowding into low-beta winners and high-beta losers. When the BAB strategy is most crowded, winners and losers remain into the extreme momentum portfolios for longer and the spread between their ranking-period returns is exacerbated. A momentum strategy that conditions on the crowding of BAB delivers a large and significant alpha. The results reveal that crowding in one strategy can have amplifying effects on the crowding in another strategy when both strategies lack fundamental anchors, are vulnerable to positive-feedback loops, and trade in stocks with overlapping characteristics.

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1 Introduction

Crowding in quantitative investment strategies has become a major concern for sophisticated traders. Crowding is the tendency of investors to implement similar strategies and trade in the same direction at the same time.[1] Crowded positions can lead to the alpha decline of a strategy (Stein (2009)) and an increase in its tail risk (Brown, Howard, and Lundblad (2019)). If there are many market participants crowded into a trade, a shock to the system may force everyone to rush to the exits at the same time, exacerbating the risk of falling prices, margin calls, and vanishing liquidity (Brunnermeier and Pedersen (2009)).[2]

Theoretical studies suggest that crowding is of special concern in unanchored investment strategies since it can generate feedback effects and cause asset prices to deviate substantially from fundamental value (e.g., Abreu and Brunnermeier (2003), Stein (2009)). Momentum is a classic example of such a positive-feedback trading strategy.[3] In the case of momentum, arbitrageurs' demand for an asset is an increasing function of its lagged returns and bets on (against) past winners (losers) can result in the prices of those securities rising (falling) further. Therefore, a high past asset return could either be a signal of positive cash flow news or a sign of overvaluation due to crowding into the strategy as a result of previous rounds of momentum trading. Since it is difficult to gauge the crowd of other arbitrageurs pursuing momentum, arbitrage activity in the strategy can become destabilizing. When destabilization is sufficiently extreme, the momentum strategy becomes vulnerable to crash risk. Avoiding momentum at times when it is crowded by other arbitrageurs may be optimal

[1] There is no formal definition of crowding in the finance literature. Crowding broadly refers to the tendency of investors to trade in a similar way in response to the same signals, leading to overlapping positions.

[2] During the Quant Meltdown of August 2007, the simultaneous liquidation of similar levered positions by quant hedge funds caused massive losses as funds reached their margin limits (Khandani and Lo (2010), Pedersen (2009)).

[3] Jegadeesh and Titman (1993) show that when portfolios are formed based on performance over the last three months to one year, past losers continue to be losers and past winners continue to be winners over the next three to twelve months. Most behavioral models attribute the existence of price momentum to either underreaction or overreaction. The debate on the driving forces behind momentum is still ongoing.

and, therefore, being able to measure the extent of crowding in the strategy is valuable.

In recent work, Lou and Polk (2022) propose a measure of crowding in momentum which is based on the outcome of the arbitrage activity in the strategy.[4] It is denoted as $CoMOM$ and measures the average pairwise correlation of factor-adjusted returns of winners and losers in the formation period. At each point in time, the return comovement within the long and short side of momentum is measured separately, and then they are averaged together to form the overall measure of momentum crowding. The basic premise of $CoMOM$ is that when arbitrageurs take positions in the long and short side of a strategy, their trades exert a simultaneous price impact on the assets and cause return comovement in the spirit of Barberis and Shleifer (2003). When $CoMOM$ is high, too many arbitrageurs have crowded into the strategy over previous rounds of trading.

Lou and Polk (2022) show that when $CoMOM$ is high in the formation period, the subsequent *long-run* profits to the momentum strategy are negative. This finding is consistent with high amounts of arbitrage activity pushing prices further away from fundamentals and eventually leading to a correction. The $CoMOM$ crowding measure, however, does not predict negative momentum profits in the *short run*. This is the case since crowding may temporarily drive prices in favour of momentum traders if they "jump on the bandwagon" before the strategy becomes "too" crowded.[5]

If high $CoMOM$ predicts that a reversal in momentum is coming further down the road but not immediately, how can an arbitrageur know that momentum has become so crowded that a sharp reversal is imminent? Can we calculate a more timely measure of momentum crowding that can be used to anticipate imminent crashes? In this paper I show that crowding in the betting-against-beta strategy (BAB) is a strong predictor of negative momentum profits in the short run (1-5 months). The degree of crowding in BAB is

[4] Hanson and Sunderam (2014) infer the amount of arbitrage capital deployed in momentum by relying on time variation in the cross section of short interest.

[5] De Long, Shleifer, Summers, and Waldmann (1990) make a similar argument.

measured by the average pairwise correlation of factor-adjusted returns of low-beta and high-beta stocks in the formation period. The measure is denoted as $CoBAR$ as in Huang, Liu, Lou, and Polk (2023). I show that following high levels of $CoBAR$, the momentum strategy yields -0.36% on average in the first month after portfolio formation. In contrast, following low levels of $CoBAR$, the momentum strategy yields 1.49% on average in the first month after portfolio formation. The difference between the two is a significant -1.85%. Crowding in the betting-against-beta strategy continues to be significantly negatively related to future momentum returns for up to 5 months after portfolio formation, controlling for $CoMOM$ (the crowding in momentum).

The predictive abilities of $CoBAR$ for momentum returns over the next month can be exploited to design a well-timed momentum strategy. In particular, if $CoBAR$ is below the 80th percentile of its distribution up to that point, the strategy buys winners and shorts losers. Otherwise, the strategy shorts winners and buys losers. I show that this conditional value-weighted momentum strategy has a significant 5-factor alpha of 1.17% per month and it performs better than the standard value-weighted momentum strategy and a strategy that conditions on $CoMOM$.

Why is crowding in one strategy relevant for the profitability of another strategy? In particular, why is $CoBAR$ a strong predictor of momentum profitability in the short run? To illustrate the intuition, assume that a single factor drives the variation in returns and the security market line is flat.[6] Assume also that the signal that arbitrageurs use for a momentum strategy is based on stocks' alphas. In this framework, high-beta stocks earn negative alphas and low-beta stocks earn positive alphas. Therefore, sorting stocks on their alphas to identify winners and losers is implicitly a reverse sort on their betas. The standard momentum strategy is based on total returns (which include alpha, a factor component, and

[6] Frazzini and Pedersen (2014) present results which are consistent with this assumption.

a residual), but the simple intuition above still holds. That is, if the security market line is flat, among the stocks that are classified as winners (losers) based on total returns, there are likely to be stocks that are also low-beta (high-beta) assets. Therefore, some winner (loser) stocks that are traded by the momentum crowd of arbitrageurs are also traded by the betting-against-beta crowd by virtue of being low-beta (high-beta) stocks.

Not only is there an overlap between the winner (loser) and low-beta (high-beta) portfolios, but the betting-against-beta strategy is another example of a positive-feedback strategy that is vulnerable to crowding. When arbitrageurs bet on low-beta stocks, the prices of these securities rise. Similarly, bets against high-beta stocks decrease their prices. According to Huang, Liu, Lou, and Polk (2023), if the underlying firms are levered, this change in price, all else equal, will result in the security's beta falling (increasing) further.[7] Therefore, when the BAB strategy becomes crowded, stocks in the extreme beta deciles are more likely to remain in these extreme groups leading to a positive-feedback loop in the beta-arbitrage strategy.

The stocks in the winner (loser) portfolio that also have low betas (high betas) will be exposed to the arbitrage activity in two strategies that are vulnerable to positive-feedback loops that destabilize prices. As the prices of low-beta winners increase further due to trading in BAB, such performance renders them stronger and they are likely to remain in the winner portfolio for the next round of momentum trading. This in turn is likely to increase their prices further as both momentum and BAB become more crowded. Similarly, as the prices of high-beta losers decrease due to trading in BAB, their performance becomes weaker and they are likely to remain in the loser portfolio for the next round of momentum trading. This in turn is likely to decrease their prices further as both momentum and BAB become more crowded. Therefore, winners with low betas and losers with high betas

[7] This follows from Proposition II of Modigliani and Miller (1958).

will be exposed to the arbitrage process of both momentum and betting-against-beta. The momentum and BAB crowds will reinforce each other, likely driving the prices of these assets further away from fundamental value.

Consistent with the positive-feedback mechanism outlined above, I show that the spread in ranking-period cumulative returns and betas between winners and losers increases when beta-arbitrage activity $CoBAR$ is high. When $CoBAR$ in the ranking period is in the highest quintile, the ranking-period return of the losers is -55.58%, while that of the winners is 93.29%, leading to a spread of 148.87%. In contrast, when $CoBAR$ is in the lowest quintile, the ranking-period return of the losers is -36.66% and that of the winners is 83.93%, leading to a spread of 120.59%. The difference in ranking-period returns between the two extreme states of $CoBAR$ is significant for both losers and winners. Furthermore, when $CoBAR$ is in the highest quintile, the ranking-period beta of the losers is 1.35 and that of the winners is 1.04, leading to a beta spread of -0.31. When $CoBAR$ is in the lowest quintile, the ranking-period beta of the losers is 1.21 and that of the winners is 1.23, for a spread of 0.02. The difference in ranking-period betas between the two extreme states of $CoBAR$ is significant for both losers and winners. All the spreads mentioned above are much smaller when crowding is measured by $CoMOM$.

As the momentum strategy is rebalanced, the composition of the winner and loser deciles changes. As a consequence of larger spreads in ranking-period returns and betas when crowding is at its highest, the turnover of winners and losers is likely to fall as investors remain invested in the same assets. I find evidence consistent with this argument. When $CoBAR$ in the ranking period is in the highest quintile, the winners (losers) in that ranking period have been in the winner (loser) portfolio on average for 4.42 (4.20) rebalance cycles (assuming monthly rebalancing). When $CoBAR$ in the ranking period is in the lowest quintile, the winners (losers) in that ranking period have been in the winner (loser) portfolio on average for 3.71 (3.73) rebalance cycles.

Since some winners (losers) are also low-beta (high-beta) stocks, their turnover relative to the BAB strategy also decreases when $CoBAR$ is high. When $CoBAR$ in the ranking period is in the highest quintile, the winners (losers) in that ranking period have been in the low-beta (high-beta) portfolio on average for 4.27 (3.89) rebalance cycles (assuming monthly rebalancing). When $CoBAR$ in the ranking period is in the lowest quintile, the winners (losers) in that ranking period have been in the low-beta (high-beta) portfolio on average for 3.00 (2.63) rebalance cycles.

The lower turnover associated with the BAB strategy creates the potential for more extreme deviations from fundamentals as the same stocks keep showing up in the portfolios of arbitrageurs. Therefore, $CoBAR$ emerges as a more potent signal of an impending correction in the prices of winners and losers than $CoMOM$. When $CoMOM$ is decomposed into two orthogonal components such that one is related to $CoBAR$ and the other is not, only the component related to $CoBAR$ has significant predictive ability for negative momentum profits in the short run. More specifically, going from a low to a high rank for the component related to $CoBAR$, the odds of observing a negative momentum return in the first month after portfolio formation increase 4.5 times.

This paper contributes to the empirical literature on crowding. The notion that momentum crowding affects the future profitability of the strategy has been studied before in the literature (e.g., Hanson and Sunderam (2013) and Lou and Polk (2022)). However, the effect of crowding in one strategy on the profitability of a different strategy has not received much attention. The innovation in this paper is to show that crowding in one strategy can have an amplifying effect on the crowding of another strategy. This is the case when there is overlap between the stocks that attract arbitrage activity in the two strategies. Furthermore, if the strategies lack a fundamental anchor, arbitrageurs have a hard time knowing when to stop trading and this leads to positive-feedback trading loops that can result in painful price corrections. The results in the paper increase our understanding of the time-series dynamics of momentum and have implications for the risk management of the strategy.

This paper also contributes to the literature on momentum crashes. I show that impending momentum crashes in the first month after portfolio formation are predictable by the crowding in the BAB strategy. This has implications for designing more robust momentum strategies that do not expose investors to significant drawdowns compared to unconditional momentum strategies.

The rest of the paper is organized as follows. Section 2 summarizes the related literature. Section 3 explains the intuition behind the overlap between ranking-period cumulative returns and betas. Section 4 describes the data, the empirical methodology, and the main results related to momentum predictability. Section 5 elaborates on the positive-feedback loop mechanism behind $CoBAR$, and Section 6 concludes.

2 Related Literature

One theoretical foundation behind the importance of crowding by arbitragers in the momentum strategy comes from the work of Hong and Stein (1999) and Stein (2009). In Hong and Stein (1999), newswatchers underreact to private signals about stock fundamentals due to slow information diffusion. When the market consists entirely of newswatchers, prices adjust slowly to new information leading to momentum as a result of underreaction. The continuation in returns creates an arbitrage opportunity for momentum traders who base their demand for stocks on recent price changes. As momentum traders profit from the newswatchers' underreaction, price movements accelerate in the direction of fundamentals. As more momentum traders join, prices overshoot long-run equilibrium values, which leads to overreaction and price correction. Hong and Stein (1999) predict that momentum investing will lose money late in the trading cycle when prices have already overshot equilibrium values as a result of many momentum traders crowding into the strategy.

Stein (2009) points out that if arbitrageurs condition their stock demand on past returns, such a momentum strategy lacks a fundamental anchor. For example, a low past return could mean that the stock received bad news and, given that newswatchers underreact to information, arbitrageurs should bid down the stock price. On the other hand, a low past return could mean that other arbitrageurs have already bid the stock price down to the extent that the initial mispricing is overcorrected. From an individual arbitrageur's perspective, implementing a momentum strategy carries the risk of getting into the strategy when it is crowded and prices are about to revert to fundamentals. If arbitrageurs are forced to withdraw capital from the momentum strategy, their collective unwinding of positions can lead to momentum crashes.

The key feature of Stein's (2009) model is that the stock price (i.e., momentum signal) is not necessarily a summary statistic for fundamental value since it is also influenced by the level of arbitrage activity in the stock. Stein (2009) concludes that if a given price realization reflects an increase in arbitrage activity, then each individual arbitrageur would be better off taking the opposite position in the stock.[8]

Several empirical studies examine whether the crowding problem discussed in Stein (2009) affects the time-series variation of momentum profitability. Lou and Polk (2022) derive a measure of arbitrage activity in momentum based on residual return correlations among typical momentum strategy stocks. They show that when their measure indicates that arbitrageurs are crowded into the strategy, momentum tends to crash and revert in

[8] In Stein (2009), each arbitrageur is uncertain about the number of other arbitrageurs who are active in the same strategy. In Abreu and Brunnermeier (2002), each arbitrageur is uncertain about when others will act on the same strategy.

the long run after portfolio formation. Hanson and Sunderam (2013) infer the amount of arbitrage capital deployed in momentum by relying on time variation in the cross section of short interest. They show that higher levels of short arbitrage capital in momentum are negatively related to the profitability of the momentum strategy.

The theoretical underpinning for the positive-feedback mechanism embedded in momentum comes from models of positive-feedback trading. In these models, positive-feedback traders exert price pressure as they buy past winners and sell past losers, thereby generating initial momentum and subsequent reversal in the cross section of stocks. For example, De Long et al. (1990) show that rational speculators have the incentive to front run positive-feedback traders in order to stimulate a price increase (decline) and take advantage of their subsequent trading.[9] If there is good news today, rational traders buy and push the price higher because feedback traders are willing to take up the position at a higher price in the next period. Therefore, the incentive to ride the bubble stems from predictable feedback trader demand.

In Barberis and Shleifer (2003), investors with extrapolative beliefs behave like positive-feedback traders, buying (selling) styles that have performed well (poorly) in the past. Eventually, after several periods of extrapolation, the price of an asset moves significantly above (below) its fundamental value. This is a sign that extrapolators have been buying (selling) it aggressively, causing it to become overpriced (underpriced). The overvaluation (undervaluation) is then followed by low (high) returns. One natural prediction of this model is that the effect on prices is stronger when there are more extrapolators in the economy. Barberis and Shleifer (2003) state that if an arbitrageur is clever enough to anticipate the behavior of extrapolators, the optimal strategy would be a momentum-like

[9] Several models that generate cross-sectional momentum feature agents that behave as positive-feedback traders, but the underlying mechanisms differ from each other. In Daniel, Hirshleifer, and Subrahmanyam (1998) the underlying mechanism is based on self-attribution bias, in Barberis et al. (1998) it is based on representativeness, while in Hong and Stein (1999) it is bounded-rationality.

strategy that rides with the crowd of extrapolators as long as their investment flows are heavily dependant on assets' past performance.

Momentum is not the only example of a positive-feedback strategy. Huang et al. (2023) point out that the betting-against beta strategy is also susceptible to positive-feedback trading. As bets on (against) low-beta (high-beta) stocks drive the prices of those securities higher (lower), their betas keep falling (increasing) further as a result of leverage. Similar to Lou and Polk (2022), Huang et al. (2023) use a measure of crowding in BAB based on residual return correlations among typical BAB strategy stocks. They show that their measure is related to the future profitability of the BAB strategy.[10]

Most studies so far examine the impact of crowding in one particular strategy on the future returns of the same strategy. This paper shows that crowding in BAB can spill over to the momentum strategy and affect its short-term profitability. The results of the empirical analysis contribute to our understanding of momentum dynamics and have important implications for trading strategy design, performance, and risk assessment.

3 Motivation for Looking at $CoBAR$

To motivate examining the connection between BAB crowding and momentum profits, in this section I present a simple intuitive argument. The key to this argument is that a cross-sectional sort on returns will overlap with a cross-sectional sort on stock betas.

To illustrate the connection between a trading signal based on returns and a signal based on betas, assume without a loss of generality that the excess market return R_M drives the

[10]Some of the alternative methods for measuring arbitrage activity include using hedge fund holdings (Brunnermeier and Nagel (2004), Griffin and Xu (2009), Cao et al. 2018) or exploiting short-selling activity in the cross section of stocks (Boehmer, Jones, and Zhang (2008), Yan (2014), Hanson and Sunderam (2014), Hwang, Liu, and Xu (2019)). Chen, Da, and Huang (2019), propose a net arbitrage trading measure that combines hedge fund holdings with short-selling information.

cross section of excess stock returns R_i:

$$R_i = \beta_i R_M + \epsilon_i. \tag{1}$$

According to the CAPM, equation (1) implies that expected excess stock returns are proportional to their betas. However, Black (1972) and Frazzini and Pedersen (2014) note that this assumption is violated under borrowing and lending constraints. In particular, investors who cannot borrow would be willing to pay for high-beta stocks because of their embedded leverage, driving up their prices and lowering their expected returns. This results in a security market line which is flatter than the one implied by equation (1).

When expected excess returns are not proportional to their betas, equation (1) becomes:

$$R_i = \alpha_i + \beta_i R_M + \epsilon_i, \tag{2}$$

where α_i is the extent of stock mispricing. In the context of an economy with borrowing and lending constraints, Frazzini and Pedersen (2014) model the intercept in equation (2) as a linear function which decreases in beta:

$$\alpha_i = \eta(\overline{\beta} - \beta_i), \tag{3}$$

where η represents the portfolio constraint for the average agent and determines the flatness of the security market line. If $\eta = 0$, the α_i intercept is zero and unconditional returns are fully described by equation (1). If $\eta > 0$, an asset's alpha decreases in its beta.[11] Combining equations (2) and (3), the return process becomes:

$$R_i = \alpha_i + \beta_i R_M + \epsilon_i = \eta(\overline{\beta} - \beta_i) + \beta_i R_M + \epsilon_i. \tag{4}$$

Therefore, if the security market line is flatter then predicted by the CAPM, a trading

[11] Frazzini and Pedersen (2014) show that this is the case in the U.S. data.

signal based on returns will be confounded with a trading signal based on betas. In other words, a cross-sectional momentum sort is inversely correlated with a cross-sectional sort on betas. Since winners tend to have higher alphas, a large fraction of them are likely to have low betas. Similarly, since losers tend to have lower alphas, a large fraction of them are likely to have high betas.[12]

The correlation between stocks' return and beta characteristics, implied by equation (4), sets up the stage for a positive-feedback trading loop that will have a strong effect on low-beta winners and high-beta losers. When arbitrageurs who trade on BAB flock to stocks with low betas, the prices of these securities will increase. If the underlying firms are levered, this increase in price, all else equal, will result in their betas falling further. Therefore, as the BAB strategy becomes crowded, stocks in the low-beta decile will remain in this decile for several rounds of portfolio rebalancing. The low-beta stocks that also happen to be past winners will be exposed to the arbitrage activity in momentum in addition to the arbitrage activity in BAB. As the prices of low-beta winners increase further due to trading in BAB, such performance renders them stronger winners and they are likely to remain in the winner portfolio for the next round of momentum trading. This in turn is likely to increase their prices further as both momentum and BAB become more crowded. Eventually, the prices of these securities will become so dislocated from fundamental value that a painful reversal must follow. A similar argument follows for high-beta losers.

The arguments above motivate using the crowding in the BAB strategy as an indicator of future momentum profits. The results in the paper show that crowding in BAB, $CoBAR$, is a better short-term predictor of momentum profits than crowding in momentum, $CoMOM$. A possible reason behind this can be traced back to the ranking-period signal used to

[12] The argument that a cross-sectional momentum sort is in effect a reverse cross-sectional sort on betas is even stronger in the case of a momentum strategy based on residual returns (i.e, the $\alpha_i + \epsilon_i$ component from Eq.(2)). Grundy and Martin (2001) show that winners and losers on the basis of total return ranking are often also winners and losers on the basis of residual return ranking. They show that the overlap between stocks ranked in the extreme deciles of total return and residual returns is around 78%.

design the two strategies. Stock returns are much less persistent than stock betas. Another way to see this is to look at the average turnover of stocks in the winner/loser and low-beta/high-beta portfolios. Assuming monthly rebalancing, the average winner (loser) stock stays in the winner (loser) decile for 3.80 (3.90) rebalance cycles on average. In contrast, the average low-beta (high-beta) stock stays in the low-beta (high-beta) decile for 6.11 (6.10) rebalance cycles on average. The lower turnover associated with the BAB strategy creates the potential for more extreme deviations from fundamentals as the same stocks keep showing up in the portfolios of arbitrageurs. Therefore, $CoBAR$ emerges as a more potent signal of an impending correction in prices.

4 Data and Methodology

4.1 Data

The sample consist of all publicly traded stocks on NYSE, AMEX, and NASDAQ that have prices higher than $5 for the period from 1968 to 2022. Stocks that are in the bottom NYSE size decile are excluded from the sample to mitigate concerns about trading in micro-cap stocks. To identify winners and losers for the momentum strategy, at the end of each month t, I sort all stocks into deciles based on their cumulative returns over the period from month $t-12$ to $t-2$. Winners (losers) are stocks with returns in the top (bottom) 10% of the distribution of cumulative returns over $t-12$ to $t-2$. At each point in time, the momentum strategy goes long in the value-weighted winner portfolio and shorts the value-weighted loser portfolio.

Table 1 reports the average returns to the momentum strategy in each of the 4 years after portfolio formation. For the first year after portfolio formation, the monthly returns are reported separately. The table shows that momentum profits are significantly positive until five months after portfolio formation. There is a significant reversal in momentum profitability at month $t+12$ after portfolio formation. Momentum profits are significantly negative on average in year 2 after formation.

4.2 Crowding Measures

After the winners and losers are identified, I calculate their residual returns in the ranking period by adjusting for the Fama-French three factors, where the betas on the factors are estimated with rolling-window regressions using daily returns over the previous year.[13] Then, I calculate average pairwise correlations for the stocks in the winner and loser portfolio, separately, using daily residual returns in the ranking period. These correlations are the basis for the momentum strategy crowding measure.

The measure of crowding in the momentum strategy is denoted as $CoMOM$ and it is calculated as the average of loser comomentum ($CoMOM_L$) and winner comomentum ($CoMOM_W$):

$$CoMOM_L = \frac{1}{N_L}\sum_{i<j} Corr(\hat{\epsilon}_i^L, \hat{\epsilon}_j^L), \tag{5}$$

$$CoMOM_W = \frac{1}{N_W}\sum_{i<j} Corr(\hat{\epsilon}_i^W, \hat{\epsilon}_j^W), \tag{6}$$

$$CoMOM = \frac{1}{2}(CoMOM_L + CoMOM_W), \tag{7}$$

where $\hat{\epsilon}_i^L$ ($\hat{\epsilon}_i^W$) is the daily residual return of stock i in the loser (winner) decile, and N_L (N_W) is the number of stocks in the loser (winner) decile.

The measure of crowding in the betting-against-beta strategy is calculated in a similar way. In particular, to identify stocks with low and high betas, at the end of each month t, I sort all stocks into deciles based on their market betas at time $t-2$.[14] Following prior literature, I calculate pre-ranking betas using daily returns in the past twelve months (with

[13] The results are robust to including industry controls by using the 30 Fama-French industry portfolios.

[14] I skip a month between the ranking period for betas and the holding period so that the beta signal of a stock is measured at the same time as its momentum signal. Results are robust if I do not skip a month since betas are persistent.

at least 126 daily observations). To account for non-synchronous trading, the pre-ranking betas are adjusted for five lags of the excess market return. After the low-beta and high-beta stocks are identified, I calculate their residual returns in the ranking period by adjusting for the Fama-French three factors, where the betas on the factors are estimated with rolling-window regressions using daily returns over the previous year.[15] Finally, I calculate average pairwise correlations for the stocks in the low-beta and high-beta portfolios, separately, using daily residual returns in the ranking period.

The beta arbitrage measure is denoted as $CoBAR$ and it is calculated as the average of low-beta arbitrage ($CoBAR_{Low\beta}$) and high-beta arbitrage ($CoBAR_{High\beta}$):

$$CoBAR_{Low\beta} = \frac{1}{N_{Low\beta}} \sum_{i<j} Corr(\hat{\epsilon}_i^{Low\beta}, \hat{\epsilon}_j^{Low\beta}), \tag{8}$$

$$CoBAR_{High\beta} = \frac{1}{N_{High\beta}} \sum_{i<j} Corr(\hat{\epsilon}_i^{High\beta}, \hat{\epsilon}_j^{High\beta}), \tag{9}$$

$$CoBAR = \frac{1}{2}(CoBAR_{Low\beta} + CoBAR_{High\beta}), \tag{10}$$

where $\hat{\epsilon}_j^{Low\beta}$ ($\hat{\epsilon}_j^{High\beta}$) is the daily residual return of stock i in the low-beta (high-beta) decile, and $N_{Low\beta}$ ($N_{High\beta}$) is the number of stocks in the low-beta (high-beta) decile.

Figure 1 plots the time series of $CoMOM$ and $CoBAR$ over the period from 1968 to 2022. The correlation between the two measures is 63%. The figure makes it clear that the two series are distinct and display independent variation of each other. It is interesting to note that as the momentum and BAB strategies have become more popular towards the second half of the sample, the overall level of crowding in the strategies has increased.[16] The level of average pairwise correlation is much higher for the BAB strategy relative to the momentum strategy. This could reflect the observation that the average stock in the extreme BAB portfolios spends a longer time in those portfolios than the average momentum stock

[15] The results are robust to including industry controls by using the 30 Fama-French industry portfolios.
[16] The results are robust to detrending both measures.

spends in the corresponding extreme deciles. That is, stocks are exposed to beta arbitrage longer than their counterparts that are exposed to momentum arbitrage. This sets them up to become highly correlated with similar stocks, i.e., it sets them up to become extremely crowded.

Once the $CoBAR$ crowding measure is computed, I examine its predictive ability for future momentum profits.

4.3 Forecasting Momentum Returns with $CoBAR$

This section examines the central question of the paper: does beta arbitrage (i.e., BAB crowding) forecast momentum profits? Table 2 reports the average returns to the momentum strategy as a function of lagged $CoBAR$. All months in the sample are ranked into five groups based on $CoBAR$ calculated during the ranking period for momentum. State "High" ("Low") corresponds to the highest (lowest) quintile of $CoBAR$ values. Average momentum returns are reported for each of the 4 years after portfolio formation, following different $CoBAR$ states. For the first year after portfolio formation, I report each of the monthly returns separately. "High-Low" is the difference in returns across high and low $CoBAR$ ranks.

Panel A of Table 2 shows that following high $CoBAR$, the momentum strategy yields -0.36% on average in the first month after portfolio formation. In contrast, following low $CoBAR$, the momentum strategy yields 1.49% on average in the first month after portfolio formation. The difference between the two is a significant -1.85%. This difference continues to be significant for 5 months after portfolio formation. This reveals that high $CoBAR$ has an immediate negative effect on momentum profitability. It is interesting to note that (with the exception of month $t+8$), average momentum returns are negative for all months during the first year after portfolio formation following high $CoBAR$ states. The negative relation between high $CoBAR$ and momentum profitability in the long run does not appear to be significant.

Panel B of Table 2 shows momentum profitability following different ranks for $CoMOM$. The results reveal that crowding in the momentum strategy is associated with negative momentum returns in the long run but not in the short run. This is consistent with previous results reported in Luo and Polk (2022).

Figure 3 graphically shows the patterns reported in Table 2. The figure plots the cumulative buy-and-hold returns to the momentum strategy in the 4 years after portfolio formation conditional on low $CoBAR$ or high $CoBAR$, and conditional on low $CoMOM$ or high $CoMOM$. The figure shows that the cumulative momentum return is positive following low $CoBAR$ or low $CoMOM$. Following high $CoBAR$, the cumulative momentum return is negative, reaching the lowest point around month $t + 20$. Following high $CoMOM$, the cumulative momentum return is initially positive and it turns negative in month $t + 17$, reaching the lowest point around month $t + 22$. Overall, the figure shows that high $CoBAR$ is associated with an immediate decline in momentum profitability, while high $CoMOM$ still predicts positive momentum returns in the short term.

Since $CoBAR$ and $CoMOM$ are positively correlated, I use time-series regressions to examine whether high $CoBAR$ is negatively associated with momentum profitability, controlling for $CoMOM$. I also control for the cumulative market return and volatility in the momentum ranking period. Table 3 reports the results from the predictive regressions. The dependent variable in each regression is the time series of momentum returns in different periods after portfolio formation. The dependent variables are $CoBAR$, $CoMOM$, the cumulative market return $MktRet$ and volatility $MktVol$, all of which are estimated during the momentum ranking period. The results show that high values of $CoBAR$ significantly predict negative momentum returns in months 1-5 after portfolio formation. Similar results hold for quarters 1 and 2 and year 1 after portfolio formation. As expected, the R^2s of the predictive models for monthly returns are very low, however, they increase for quarterly and yearly returns. $CoMOM$ becomes significantly associated with negative momentum returns around 8 months after portfolio formation.

Overall, the predictive regressions in Table 3 reinforce the conclusion that high levels of crowding in the BAB strategy are significantly associated with negative momentum returns in the short term after portfolio formation. This conclusion is robust to controlling for the crowding in the momentum strategy and the performance of the market return during the formation period.

4.4 Economic Significance of the Forecasting Abilities of $CoBAR$

One way to measure the economic significance of the results reported in Table 3 is by designing a trading strategy that uses $CoBAR$ as a conditioning variable. In order to assure that the strategy is implementable in real time and free of look-ahead bias, I define high $CoBAR$ states as months in which $CoBAR$ is above the 80th percentile of its 36-month rolling distribution up to that point. The conditional strategy then buys winners and shorts losers when $CoBAR$ is not high and holds them for a month. When $CoBAR$ is high, the conditional strategy shorts winners and buys losers instead. Therefore, this strategy, denoted as MOM_{CoBAR}, is designed to take advantage of the observation that $CoBAR$ is a significant predictor of momentum reversal in the short run.

For the sake of comparison, I also examine a trading strategy that uses $CoMOM$ as a conditioning variable. I define high $CoMOM$ states as months in which $CoMOM$ is above the 80th percentile of its 36-month rolling distribution up to that point. Then, the strategy MOM_{CoMOM} buys winners and shorts losers when $CoMOM$ is not high and holds them for a month. When $CoMOM$ is high, MOM_{CoMOM} shorts winners and buys losers.

Table 4 reports the alphas of three momentum strategies: MOM_{CoBAR}, MOM_{CoMOM}, and the regular unconditional momentum strategy MOM. The alpha is computed relative to the Fama-French (2015) 5-factor model augmented with the betting-against-beta factor of Frazzini and Pedersen (2014). The alpha of MOM_{CoBAR} is 1.17% per month and significant. This is higher than the alphas of MOM_{CoMOM} and MOM which are 0.83% and 0.81%, respectively.

Table 4 also reports monthly summary statistics for the three strategies. All three have similar volatilities, but the Sharpe ratio of MOM_{CoBAR} is the highest given that it has the highest average return. It is interesting to note that the conditional momentum strategies have much lower negative skewness than the unconditional momentum strategy. The three strategies are not highly correlated.

Overall, the results in Table 4 reveal that a simple momentum strategy that conditions on the state of crowding in the BAB strategy delivers a large and significant alpha in the first month after portfolio formation. The performance of this well-timed momentum strategy is superior to the performance of the standard unconditional momentum strategy.

4.5 Forecasting Momentum Crashes with $CoBAR$

Daniel and Moskowitz (2016) show that the momentum strategy is characterized by infrequent episodes of negative returns that are clustered in time. They argue that these momentum crashes are forecastable. Daniel, Jagannathan, and Kim (2019) study the empirical distribution of momentum and conclude that it is left skewed and significantly leptokurtic. They also argue that severe momentum losses are predictable. Crowded trading by arbitrageurs may be one of the contributing factors to the crashes experienced by the momentum strategy. If arbitrageurs are forced to cover their positions in an asset when it is crowded, their collective unwinding of levered positions can lead to abrupt negative returns. In this section, I study whether high values of $CoBAR$ are associated with negative momentum returns in the near future.

Table 5 reports what percent of momentum returns after portfolio formation are negative, following different values for $CoBAR$ (Panel A) and $CoMOM$ (Panel B). Panel A shows that when $CoBAR$ is in its highest quintile, 51% of momentum returns one month after portfolio formation are negative. In contrast, when $CoBAR$ is in its lowest quintile, 34% of momentum returns one month after portfolio formation are negative. The difference between the two states in terms of frequency of negative returns is 17% and it is significant. Almost identical results hold for the incidence of negative momentum returns two months after portfolio formation. In the case of three months after portfolio formation, the difference between the negative returns frequency following high and low $CoBAR$ states is not significant. The results suggest that high $CoBAR$ has significant predictive abilities for negative momentum returns immediately after portfolio formation.

Panel B of Table 5 examines what percent of momentum returns after portfolio formation are negative, following different values for $CoMOM$. For the first month after portfolio formation, high $CoMOM$ is less frequently followed by negative momentum returns than low $CoMOM$. This relation goes against $CoMOM$ being able to predict imminent momentum crashes.

Overall, the results in Table 5 show that $CoBAR$ is not only a strong predictor of momentum profitability in the short run, but it is also able to anticipate impending momentum crashes. This is a valuable signal for arbitrageurs who may want to stay on the sidelines of momentum while its crash risk is high.

5 Positive-feedback Loop behind $CoBAR$

The previous sections show that crowding in the betting-against-beta strategy is a powerful predictor of momentum profitability in the near future. In this section, I dig deeper to examine whether the positive-feedback nature of BAB is behind the evidence documented above.

Huang, Liu, Lou, and Polk (2023) point out that BAB has an endogenous positive-feedback mechanism embedded into the strategy. In particular, the collective crowding of arbitrageurs into the strategy amplifies the stock characteristics used as trading signals. Bets in favor of low-beta stocks will increase their prices and as a consequence further decrease their betas. In contrast, bets against high-beta stocks will decrease their prices and further increase their betas. Such a self-reinforcing loop will decrease the turnover of stocks in the extreme beta portfolios as arbitrageurs remain invested in the same assets or keep shorting the same assets.

To the extent that past winners (losers) tend to have lower (higher) betas, as discussed in Section 2, they will be affected by the positive-feedback trading mechanism of the BAB strategy, in addition to a similar mechanism in the momentum strategy. As the prices of low-beta winners are pushed up by beta arbitrage, their momentum ranking characteristic becomes stronger and they are likely to remain in the winner portfolio for the next round of momentum trading. Overall, when BAB has become too crowded, we expect to observe higher ranking-period returns for the winner portfolio and a lower turnover of the stocks in the portfolio.

Table 6 reports the ranking-period returns (Panel A) and betas (Panel B) for the winner and loser deciles of the momentum strategy, following different states of $CoBAR$ and $CoMOM$. In Panel A, when $CoBAR$ in the ranking period is in the highest quintile, the ranking-period return of the losers is -55.58%, while that of the winners is 93.29%, leading to a spread of 148.87%. In contrast, when $CoBAR$ is in the lowest quintile, the ranking-period return of the losers is -36.66% and that of the winners is 83.93%, leading to a spread of 120.59%. The difference in ranking-period returns between the two extreme states of $CoBAR$ is significant for both losers and winners.

The pattern of the results is similar in the case of $CoMOM$, however, the magnitude of ranking-period returns and the spread in those returns following high $CoMOM$ are much smaller.

Furthermore, Panel B of Table 6 shows that when $CoBAR$ is in the highest quintile, the ranking-period beta of the losers is 1.35 and that of the winners is 1.04, leading to a beta spread of -0.31. When $CoBAR$ is in the lowest quintile, the ranking-period beta of the losers is 1.21 and that of the winners is 1.23, for a spread of 0.02. The difference in ranking-period betas between the two extreme states of $CoBAR$ is significant for both losers and winners. All the spreads mentioned above are much smaller when crowding is measured by $CoMOM$.

In summary, the spread in ranking-period cumulative returns and betas between winners and losers increases when beta-arbitrage activity $CoBAR$ is high and BAB is crowded. This is consistent with a positive-feedback trading mechanism embedded into the BAB strategy.

As a consequence of larger spreads in ranking-period returns and betas when crowding is at its highest, the turnover of winners and losers within the extreme momentum portfolios is likely to fall as investors remain invested in the same assets. I find evidence consistent with this argument.

I calculate the turnover within the winner and loser portfolios in the following way. I assume monthly rebalancing and at each portfolio-formation month I trace back whether a stock was in the winner (loser) portfolio at the previous portfolio-formation month. I keep going back until the stock drops out of the winner (loser) portfolio at one of the past portfolio-formation months. I average the length of time each stocks spends in the winner (loser) portfolio across all stocks in a decile and that becomes the measure of turnover in the corresponding decile.

Table 7 reports the results of this analysis, following different states of crowding for $CoBAR$ and $CoMOM$. Panel A shows that when $CoBAR$ in the ranking period is in the highest quintile, the average winner (loser) stock in that ranking period has been in the winner (loser) portfolio on average for 4.42 (4.20) rebalance cycles. When $CoBAR$ in the ranking period is in the lowest quintile, the average winner (loser) in that ranking period has been in the winner (loser) portfolio on average for 3.71 (3.73) rebalance cycles. The difference between the high and low $CoBAR$ states are significant for both winners and losers. These results show that winners (losers) tend to spend more time in the winner (loser) portfolio as the crowding in BAB intensifies.

Panel B of Table 7 shows that when $CoBAR$ in the ranking period is in the highest quintile, the average winner (loser) stock in that ranking period has been in the low-beta (high-beta) portfolio on average for 4.27 (3.89) rebalance cycles. When $CoBAR$ in the ranking period is in the lowest quintile, the average winner (loser) in that ranking period has been in the low-beta (high-beta) portfolio on average for 3.51 (3.43) rebalance cycles. The difference between the high and low $CoBAR$ states are significant for both winners and losers. Therefore, winners (losers) tend to spend more time in the low-beta (high-beta) portfolio as the crowding in BAB intensifies.

6 Decomposing $CoMOM$

The results so far show that $CoBAB$ is a more robust predictor of impending momentum crashes than $CoMOM$. Since there is an overlap between the characteristics of stocks that enter into the extreme portfolios of the momentum and betting-against-beta strategy, the $CoBAB$ and $CoMOM$ measures are also positively related to each other. In this section, I decompose the time series of $CoMOM$ into a component related to $CoBAB$ and an orthogonal component. The goal is to examine in more detail whether the overlap between the two strategy is the driving force behind the predictability of momentum profits. In particular, I examine the model:

$$CoMOM_t = a + b * CoBAB_t + u_t, \tag{11}$$

where $\hat{a} + \hat{b} * CoBAB_t$ is the component of momentum crowding related to crowding in the BAB strategy (i.e., the overlap between the two strategies) and u_t is the component of momentum crowding which is orthogonal to BAB crowding. I denote the two components, $CoMOM_{pred}$ and $CoMOM_{resid}$, respectively.

Table 8 reports the average returns to the momentum strategy as a function of lagged $CoMOM_{pred}$ and $CoMOM_{resid}$. All months in the sample are ranked into five groups based on $CoMOM_{pred}$ or $CoMOM_{resid}$ calculated during the ranking period for momentum. State "High" ("Low") corresponds to the highest (lowest) quintile of $CoMOM_{pred}$ ($CoMOM_{resid}$) values. Average momentum returns are reported for each of the 4 years after portfolio formation, following different $CoMOM_{pred}$ ($CoMOM_{resid}$) states. For the first year after portfolio formation, I report each of the monthly returns separately. "High-Low" is the difference in returns across high and low $CoMOM_{pred}$ ($CoMOM_{resid}$) ranks.

Panel A of Table 8 shows that $CoMOM_{pred}$ has an immediate negative effect on momentum profitability. With the exception of month $t+8$, average momentum returns are negative for all months during the first year after portfolio formation following high $CoMOM_{pred}$ states.

Panel B of Table 8 shows momentum profitability following different ranks for $CoMOM_{resid}$. The results reveal that high levels of crowding in the momentum strategy which is unrelated to crowding in the BAB strategy is associated with significantly higher momentum returns in the three months after portfolio formation. Following high levels of $CoMOM_{resid}$, momentum returns become negative in the long run, but not immediately after portfolio formation.

Overall, the results in Table 8 show that the component of $CoMOM$ which is related to $CoBAB$ is associated with an immediate decline in momentum profitability. In contrast, the component of $CoMOM$ which is orthogonal to $CoBAB$ still predicts positive momentum returns in the short term.

To examine more specifically the ability of the two components $CoMOM_{pred}$ and $CoMOM_{resid}$ to predict momentum crashed in the first month after portfolio formation, I estimate the following logistic model:

$$P(I_{mom_{t+1}} = 1) = \frac{1}{1 + exp(-(\beta_0 + \beta_1 * Rank^{CoMOM_{pred_t}} + \beta_2 * Rank^{CoMOM_{resid_t}}))}, \quad (12)$$

where $I_{mom_{t+1}}$ is an indicator variable equal to 1 when momentum profits in the first month after portfolio formation are negative, $Rank^{CoMOM_{pred_t}}$ is the rank of the $CoMOM_{pred}$ component of the momentum crowd measure, and $Rank^{CoMOM_{pred_t}}$ is the rank of the $CoMOM_{resid}$ component of the momentum crowd measure. The ranks are based on quintile sorts for the corresponding variables.

Table 9 reports the results from estimating model (12). The table shows that the coefficient β_1 is significantly positive, while the coefficient β_2 is negative but not significantly different from zero. This implies that higher values for the $CoMOM_{pred}$ component of the momentum crowd are associated with higher probability of observing negative momentum returns in the first month after portfolio formation. Based on the odds ratio calculations, going from a low rank for $CoMOM_{pred}$ to a high rank for $CoMOM_{pred}$, the odds of observing a negative momentum return in the first month after portfolio formation increase 4.5 times ((5-1)*1.12). The $CoMOM_{resid}$ component of the momentum crowd is not associated with increased probability of observing negative momentum returns in the short run. Overall, the results in Table 9 reveal that the component of the momentum crowd which is related to overcrowding in the BAB strategy is the one that is responsible for predicting negative momentum returns in the short run.

7 Conclusion

Momentum and betting-against-beta are two distinct investment strategies. However, in an economy with a flat security market line, there likely exists an overlap between winners and low-beta stocks and an overlap between losers and high-beta stocks. This observation sets the stage for the arbitrage activity in one strategy to affect the profitability of the other. In particular, this paper shows that high levels of crowding in BAB have a significantly negative impact on momentum profits in the first few months after portfolio formation. Furthermore, when the BAB strategy is the most crowded, more than 50% of momentum returns in the first month after portfolio formation are negative. The crowding in the momentum strategy itself cannot achieve such impressive predictive abilities for momentum profits in the short run.

What makes $CoBAR$ more successful at short-term momentum predictability than $CoMOM$? The answer is related to the ranking-period stock characteristics used by the BAB strategy. In particular, the ranking-period betas used to design the BAB strategy are much more persistent than the ranking-period returns used to design the momentum strategy. This is reflected in the average low-beta (high-beta) stock staying in the low-beta (high-beta) decile for 6.11 (6.10) rebalance cycles on average, while the average winner (loser) stays in the winner (loser) decile for 3.80 (3.90) rebalance cycles on average. The lower turnover associated with the BAB strategy creates the potential for more extreme deviations from fundamentals to build up as arbitrageurs crowd into the strategy. This is particularly important in the case of winner (loser) stocks that happen to also be present in the low-beta (high-beta) deciles. These stocks spend more time in the corresponding extreme portfolios of momentum as a result of their prices going further up (down) due to positive-feedback trading by momentum and BAB arbitrageurs. The stocks that reside longer in the extreme momentum deciles due to the positive-feedback loop eventually expose the strategy to painful reversals when crowding becomes unsustainable. The results in this paper reveal that high levels of crowding in BAB correctly anticipates these episodes of overcrowding and impending momentum crashes. Overall, the results provide evidence that arbitrage activity in one strategy can have destabilizing effect on the profits of another strategy, especially when both strategies are characterized by feedback trading loops.

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Table 1. **Average Momentum Returns**

This table reports the average returns (in %) and t-statistics to the value-weighted momentum strategy in each of the 4 years after portfolio formation. For the first year after portfolio formation, the monthly returns are reported separately. At the end of each month, all stocks are sorted into deciles based on their lagged 11-month cumulative returns (skipping the most recent month). Stocks with prices below $5 and/or that are in the bottom NYSE size decile are excluded from the sample. The momentum strategy goes long in the value-weighted winner portfolio and shorts the value-weighted loser portfolio. The sample period is from 1968 to 2022.

	Average Momentum Returns	
Period	Mean	t-stat
t+1	0.95	3.54
t+2	0.83	3.26
t+3	0.69	2.84
t+4	0.54	2.17
t+5	0.55	2.21
t+6	0.23	0.92
t+7	0.23	0.95
t+8	0.13	0.51
t+9	-0.18	-0.78
t+10	-0.40	-1.72
t+11	-0.42	-1.86
t+12	-0.78	-3.24
Y2	-0.44	-7.00
Y3	-0.11	-1.84
Y4	-0.07	-1.35

Table 2. **Forecasting Momentum Returns with $CoBAR$ and $CoMOM$**

This table reports the average returns to the momentum strategy as a function of lagged $CoBAR$ (Panel A) or lagged $CoMOM$ (Panel B). At the end of each month, all stocks are sorted into deciles based on their lagged 11-month cumulative returns (skipping the most recent month). Stocks with prices below $5 and/or that are in the bottom NYSE size decile are excluded from the sample. All months are classified into five groups based on $CoBAR$ or $CoMOM$. The table reports the average returns to the value-weighted momentum strategy in each of the 4 years after portfolio formation, following low to high $CoBAR$ or $CoMOM$. For the first year after portfolio formation, the monthly returns are reported separately. "High-Low" is the difference in returns across high and low $CoBAR$ or $CoMOM$ ranks. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively. Significance is computed based on Newey-West standard errors. The sample period is from 1968 to 2022.

Panel A: $CoBAR$ Rank						
Period	Low	2	3	4	High	High-Low
t+1	1.49	0.49	1.17	1.94	-0.36	-1.85***
t+2	1.14	1.22	0.51	1.75	-0.46	-1.60***
t+3	0.78	1.36	0.84	0.52	-0.34	-1.12**
t+4	0.71	0.57	0.79	0.84	-0.30	-1.02**
t+5	0.98	0.46	0.61	1.11	-0.40	-1.38***
t+6	0.39	-0.33	0.25	0.87	-0.02	-0.40
t+7	0.58	-0.44	0.63	0.66	-0.26	-0.85*
t+8	0.44	-0.82	0.73	0.17	0.12	-0.31
t+9	0.07	-0.71	-0.38	0.72	-0.57	-0.64*
t+10	-0.39	-0.70	-0.35	-0.13	-0.41	-0.02
t+11	-0.15	-1.13	0.10	-0.67	-0.29	-0.13
t+12	-0.14	-1.17	-0.99	-1.14	-0.48	-0.34*
Y2	-0.04	-0.48	-0.70	-0.90	-0.12	-0.08
Y3	-0.31	0.15	-0.39	-0.29	0.33	0.64**
Y4	-0.15	-0.05	0.03	-0.23	0.04	0.19
N	131	132	132	132	131	

Panel B: $CoMOM$ Rank						
Period	Low	2	3	4	High	High-Low
t+1	0.86	1.30	1.54	-0.10	1.13	0.27
t+2	0.74	1.51	0.87	0.28	0.75	0.01
t+3	1.22	0.58	0.82	0.41	0.46	-0.76**
t+4	0.92	0.64	0.77	0.13	0.23	-0.69**
t+5	1.18	0.32	0.17	0.60	0.48	-0.71**
t+6	0.97	0.13	-0.11	0.26	-0.13	-1.09**
t+7	0.67	0.82	-0.01	0.06	-0.42	-1.10**
t+8	0.61	0.10	0.66	-0.91	0.18	-0.43
t+9	0.06	0.11	0.26	-1.30	-0.04	-0.10
t+10	-0.34	0.08	-0.40	-1.38	0.09	0.42
t+11	-0.25	-0.23	-0.13	-1.55	0.08	0.32
t+12	-0.60	-0.54	-1.25	-0.98	-0.52	0.08
Y2	-0.18	-0.43	-0.89	-0.16	-0.57	-0.39*
Y3	-0.11	-0.60	-0.70	0.48	0.51	0.62**
Y4	-0.01	-0.37	-0.17	-0.03	0.33	0.35
N	131	132	132	132	131	

Table 3. **Time-series Predictive Regressions with $CoBAR$ and $CoMOM$**

This table reports the results of time-series regressions in which the dependent variable is the time series of momentum returns in different periods after portfolio formation. For example, mom_{t+1}, is the momentum return one month after portfolio formation, mom_{q1} is the average momentum return in the first quarter after portfolio formation, mom_{y1} is the average momentum return in the first year after portfolio formation, etc. The dependent variables are $CoBAR$, $CoMOM$, the cumulative market return $MktRet$ and volatility $MktVol$, all of which are estimated during the momentum ranking period. The Newey-West t-statistics are below the coefficients. The sample period is from 1968 to 2022.

Panel A: Monthly Returns

Dep Var.:	mom_{t+1}	mom_{t+2}	mom_{t+3}	mom_{t+4}	mom_{t+5}	mom_{t+6}	mom_{t+7}	mom_{t+8}	mom_{t+9}	mom_{t+10}	mom_{t+11}	mom_{t+12}
Intercept	0.03	0.03	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.00	0.00	0.00
	[2.97]	[3.02]	[2.60]	[2.44]	[1.86]	[1.16]	[1.51]	[1.51]	[0.83]	[0.22]	[-0.56]	[0.00]
$CoBAR$	-0.44	-0.44	-0.54	-0.38	-0.46	-0.23	-0.09	0.24	0.31	0.15	0.16	0.17
	[-2.64]	[-2.81]	[-3.54]	[-2.45]	[-2.99]	[-1.44]	[-0.61]	[1.54]	[2.10]	[1.00]	[1.08]	[1.13]
$CoMOM$	0.57	0.54	0.95	0.51	0.86	0.61	0.04	-0.93	-1.04	-0.15	-0.09	-0.20
	[1.22]	[1.23]	[2.25]	[1.18]	[2.01]	[1.40]	[0.10]	[-2.16]	[-2.53]	[-0.36]	[-0.22]	[-0.48]
$MktRet$	0.01	0.00	0.01	0.00	0.00	0.00	0.00	-0.02	-0.02	-0.01	-0.01	-0.02
	[0.39]	[0.04]	[0.55]	[0.13]	[0.10]	[0.13]	[0.04]	[-0.92]	[-1.31]	[-0.76]	[0.86]	[-1.02]
$MktVol$	-0.35	-0.32	-0.30	-0.28	-0.19	-0.20	-0.20	-0.15	-0.07	-0.15	-0.01	-0.19
	[-1.97]	[-1.89]	[-1.89]	[-1.72]	[-1.18]	[-1.22]	[-1.24]	[-0.92]	[-0.44]	[-0.95]	[-0.08]	[-1.18]
$Adj.R^2$	0.02	0.02	0.02	0.01	0.01	0.00	0.00	0.00	0.01	0.00	0.00	0.00
N	647	647	647	647	647	647	647	647	647	647	647	647

Panel B: Quarterly and Annual Returns

Dep Var.:	mom_{q1}	mom_{q2}	mom_{q3}	mom_{q4}	mom_{y1}	mom_{y2}	mom_{y3}	mom_{y4}
Intercept	0.03	0.02	0.01	0.00	0.01	0.01	0.00	0.00
	[6.12]	[3.71]	[2.20]	[0.20]	[6.63]	[3.19]	[1.64]	[0.00]
$CoBAR$	-0.43	-0.30	0.16	0.12	-0.13	0.09	-0.01	-0.08
	[-4.72]	[-3.18]	[1.80]	[1.39]	[-3.23]	[2.27]	[-0.26]	[-2.31]
$CoMOM$	0.65	0.54	-0.69	-0.15	0.17	-0.17	0.40	0.39
	[2.77]	[2.23]	[-3.02]	[-0.66]	[1.78]	[-1.63]	[4.15]	[4.36]
$MktRet$	0.00	0.00	-0.01	-0.02	-0.01	-0.02	-0.02	0.00
	[0.51]	[0.38]	[-1.18]	[-2.12]	[-1.22]	[-3.81]	[-5.50]	[-1.08]
$MktVol$	-0.44	-0.31	-0.13	-0.14	-0.26	-0.23	-0.14	-0.06
	[-4.69]	[-3.20]	[-1.42]	[-1.51]	[-5.98]	[-5.73]	[-3.67]	[-1.69]
$Adj.R^2$	0.07	0.03	0.01	0.01	0.07	0.06	0.12	0.04
N	649	649	649	649	622	622	622	622

Table 4. **Conditional Momentum Strategies**

This table reports the alphas (in %), factor exposures, summary statistics, and correlations for three momentum strategies. Mean and standard deviation (SD) are reported in % per month. The MOM_{CoBAR} and MOM_{CoMOM} momentum strategies use $CoBAR$ and $CoMOM$ as a timing signal, respectively. High $CoBAR$ ($CoMOM$) states are defined as months in which $CoBAR$ ($CoMOM$) is above the 80th percentile of its 36-month rolling distribution up to that point. The conditional strategy MOM_{CoBAR} buys winners and shorts losers when $CoBAR$ is not high and holds them for a month. When $CoBAR$ is high, MOM_{CoBAR} shorts winners and buys losers. The conditional strategy MOM_{CoMOM} buys winners and shorts losers when $CoMOM$ is not high and holds them for a month. When $CoMOM$ is high, MOM_{CoMOM} shorts winners and buys losers. MOM is the regular unconditional momentum strategy. All strategies are value-weighted. Alphas and factor exposures are derived from the Fama-French (2015) 5-factor model, augmented with the betting-against-beta (BAB) factor of Frazzini and Pedersen (2014). The sample period is from 1968 to 2022.

Dep. Var.:	MOM_{CoBAR}	MOM_{CoMOM}	MOM
α	1.17	0.83	0.81
	[4.07]	[2.85]	[3.13]
$MktRF$	-0.08	-0.20	-0.40
	[-1.23]	[-3.01]	[-6.67]
SMB	-0.29	-0.16	-0.22
	[-2.92]	[-1.55]	[-2.40]
HML	-0.19	0.00	-0.92
	[-1.53]	[0.03]	[-8.29]
RMW	-0.70	-0.12	0.08
	[-5.06]	[-0.83]	[0.63]
CMA	0.35	-0.44	0.52
	[1.78]	[-2.24]	[2.97]
BAB	0.07	-0.06	0.54
	[0.79]	[-0.61]	[6.64]
$Adj.R^2$	0.06	0.03	0.23
N	622	622	622
Mean	0.96	0.47	0.91
SD	6.90	6.95	6.91
Sharpe	0.14	0.07	0.13
Skew	-0.15	-0.16	-0.85
Correlations	MOM_{CoBAR}	MOM_{CoMOM}	MOM
MOM_{CoBAR}	1		
MOM_{CoMOM}	-0.04	1	
MOM	0.38	0.07	1

Table 5. **Forecasting Negative Momentum Returns with $CoBAR$ and $CoMOM$**

This table reports the percent of negative returns to the momentum strategy as a function of lagged $CoBAR$ (Panel A) or lagged $CoMOM$ (Panel B). At the end of each month, all stocks are sorted into deciles based on their lagged 11-month cumulative returns (skipping the most recent month). Stocks with prices below $5 and/or that are in the bottom NYSE size decile are excluded from the sample. All months are classified into five groups based on $CoBAR$ or $CoMOM$. The table reports the percent of negative returns to the value-weighted momentum strategy in each of the 3 months after portfolio formation, following low to high crowding measures. "High-Low" is the difference in percent of negative returns across high and low crowding ranks. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively. Significance is computed based on Newey-West standard errors. The sample period is from 1968 to 2022.

	Panel A: $CoBAR$ Rank					
Period	Low	2	3	4	High	High-Low
t+1	0.34	0.42	0.39	0.36	0.51	0.17**
t+2	0.34	0.39	0.42	0.36	0.50	0.16**
t+3	0.42	0.42	0.42	0.45	0.47	0.05
N	131	132	132	132	131	

	Panel B: $CoMOM$ Rank					
Period	Low	2	3	4	High	High-Low
t+1	0.42	0.35	0.33	0.54	0.37	-0.05
t+2	0.40	0.32	0.40	0.45	0.44	0.03
t+3	0.43	0.46	0.42	0.45	0.42	-0.01
N	131	132	132	132	131	

Table 6. **Ranking-period Returns and Betas for Momentum Portfolios**

This table reports the ranking-period returns (Panel A) and betas (Panel B) for the winner and loser deciles of the momentum strategy as a function of the lagged crowding measures $CoBAR$ and $CoMOM$. At the end of each month, all stocks are sorted into deciles based on their lagged 11-month cumulative returns (skipping the most recent month). Stocks with prices below $5 and/or that are in the bottom NYSE size decile are excluded from the sample. All months are classified into five groups based on $CoBAR$ and $CoMOM$. The table reports the ranking-period returns and betas to the extreme deciles of the value-weighted momentum strategy, following low to high crowding measures. "High-Low" is the difference in ranking-period returns or betas across high and low crowding ranks. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively. Significance is computed based on Newey-West standard errors. The sample period is from 1968 to 2022.

Panel A: Ranking-period Returns						
			$CoBAR$ Rank			
Decile	Low	2	3	4	High	High-Low
Loser	-36.66	-39.81	-42.80	-43.18	-55.58	-18.92***
Winner	83.93	80.83	77.63	85.55	93.29	9.36***
			$CoMOM$ Rank			
Decile	Low	2	3	4	High	High-Low
Loser	-39.08	-41.66	-45.98	-42.24	-49.04	-9.96***
Winner	77.79	83.25	86.73	90.26	83.17	5.38**

Panel B: Ranking-period Betas						
			$CoBAR$ Rank			
Decile	Low	2	3	4	High	High-Low
Loser	1.21	1.26	1.23	1.20	1.35	0.14
Winner	1.23	1.20	1.20	1.22	1.04	-0.19
			$CoMOM$ Rank			
Decile	Low	2	3	4	High	High-Low
Loser	1.24	1.22	1.27	1.23	1.28	0.04
Winner	1.17	1.25	1.17	1.22	1.10	-0.07

Table 7. **Momentum and BAB Frequency for Momentum Portfolios**

Panel A reports the number of rebalance cycles that the average winner (loser) has spent in the winner (loser) portfolio at portfolio formation, following low to high crowding states for $CoBAR$ and $CoMOM$. This is referred to as "Momentum Turnover." Panel B reports the number of rebalance cycles that the average winner (loser) has spent in the low-beta (high-beta) portfolio at portfolio formation, following different crowding states for $CoBAR$ and $CoMOM$. This is referred to as "BAB Turnover." At the end of each month, all stocks are sorted into deciles based on their lagged 11-month cumulative returns (skipping the most recent month). Stocks with prices below $5 and/or that are in the bottom NYSE size decile are excluded from the sample. All months are classified into five groups based on $CoBAR$ and $CoMOM$. "High-Low" is the difference in number of rebalance cycles across high and low crowding ranks. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively. Significance is computed based on Newey-West standard errors. The sample period is from 1968 to 2022.

Panel A: Momentum Turnover						
			$CoBAR$ Rank			
Decile	Low	2	3	4	High	High-Low
Loser	3.73	3.72	3.87	4.03	4.20	0.47**
Winner	3.71	3.77	3.91	4.06	4.42	0.71***
			$CoMOM$ Rank			
Decile	Low	2	3	4	High	High-Low
Loser	3.85	3.82	3.89	4.01	4.09	0.24
Winner	3.65	3.78	3.96	4.00	3.75	0.10

Panel B: BAB Turnover						
			$CoBAR$ Rank			
Decile	Low	2	3	4	High	High-Low
Loser	2.63	2.74	3.04	3.01	3.89	1.26***
Winner	3.00	3.08	3.20	3.33	4.27	1.27***
			$CoMOM$ Rank			
Decile	Low	2	3	4	High	High-Low
Loser	2.84	2.84	2.85	2.96	3.43	0.59**
Winner	3.30	3.12	3.14	3.27	3.51	0.21

Table 8. **Forecasting Momentum Returns with the Predicted and Residual Components of $CoMOM$**

The $CoMOM$ measure is decomposed into a predicted and residual component using the regression: $CoMOM_t = a + b * CoBAB_t + u_t$. The predicted (residual) component of $CoMOM$ is denoted as $CoMOM_{pred}$ ($CoMOM_{resid}$). This table reports the average returns to the momentum strategy as a function of lagged $CoMOM_{pred}$ (Panel A) or lagged $CoMOM_{resid}$ (Panel B). At the end of each month, all stocks are sorted into deciles based on their lagged 11-month cumulative returns (skipping the most recent month). Stocks with prices below $5 and/or that are in the bottom NYSE size decile are excluded from the sample. All months are classified into five groups based on $CoMOM_{pred}$ or $CoMOM_{resid}$. The table reports the average returns to the value-weighted momentum strategy in each of the 4 years after portfolio formation, following low to high $CoMOM_{pred}$ or $CoMOM_{resid}$. For the first year after portfolio formation, the monthly returns are reported separately. "High-Low" is the difference in returns across high and low $CoMOM_{pred}$ or $CoMOM_{resid}$ ranks. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively. Significance is computed based on Newey-West standard errors. The sample period is from 1968 to 2022.

Panel A: $CoMOM_{pred}$ Rank

Period	Low	2	3	4	High	High-Low
t+1	1.49	0.49	1.17	1.94	-0.36	-1.85***
t+2	1.14	1.22	0.51	1.75	-0.46	-1.60***
t+3	0.78	1.36	0.84	0.52	-0.34	-1.12**
t+4	0.71	0.57	0.79	0.84	-0.30	-1.02**
t+5	0.98	0.46	0.61	1.11	-0.40	-1.38***
t+6	0.39	-0.33	0.25	0.87	-0.02	-0.40
t+7	0.58	-0.44	0.63	0.66	-0.26	-0.85*
t+8	0.44	-0.82	0.73	0.17	0.12	-0.31
t+9	0.07	-0.71	-0.38	0.72	-0.57	-0.64*
t+10	-0.39	-0.70	-0.35	-0.13	-0.41	-0.02
t+11	-0.15	-1.13	0.10	-0.67	-0.29	-0.13
t+12	-0.14	-1.17	-0.99	-1.14	-0.48	-0.34*
Y2	-0.04	-0.48	-0.70	-0.90	-0.12	-0.08
Y3	-0.31	0.15	-0.39	-0.29	0.33	0.64**
Y4	-0.15	-0.05	0.03	-0.23	0.04	0.19

Panel B: $CoMOM_{resid}$ Rank

Period	Low	2	3	4	High	High-Low
t+1	0.52	0.69	1.46	0.39	1.67	1.15**
t+2	0.31	1.30	0.76	0.51	1.27	0.95**
t+3	0.37	0.96	1.30	-0.54	1.39	1.02**
t+4	0.33	0.61	0.40	1.19	0.15	-0.18
t+5	0.71	0.81	0.20	0.39	0.65	-0.07
t+6	0.73	0.53	0.24	-0.04	-0.34	-1.07**
t+7	0.76	0.75	0.35	-0.11	-0.65	-1.41**
t+8	0.54	0.27	0.27	0.19	-0.67	-1.21**
t+9	0.13	0.14	0.35	-0.57	-1.02	-1.15**
t+10	0.01	-0.69	0.46	-1.14	-0.65	-0.66
t+11	-0.33	0.06	-0.18	-1.17	-0.52	-0.19
t+12	-0.70	-0.60	-0.55	-1.15	-0.92	-0.22
Y2	-0.43	-0.14	-0.59	-0.48	-0.61	-0.18
Y3	-0.24	-0.18	-0.57	-0.25	0.88	1.12**
Y4	-0.22	-0.14	-0.32	-0.10	0.58	0.80

Table 9. **Forecasting Negative Momentum Returns with the Predicted and Residual Components of $CoMOM$**

This table reports results from estimating the following logistic model:

$$P(I_{mom_{t+1}} = 1) = \frac{1}{1 + exp(-(\beta_0 + \beta_1 * Rank^{CoMOM_{pred_t}} + \beta_2 * Rank^{CoMOM_{resid_t}}))},$$

where $I_{mom_{t+1}}$ is an indicator variable equal to 1 when momentum profits in the first month after portfolio formation are negative, $Rank^{CoMOM_{pred_t}}$ is the rank of the $CoMOM_{pred}$ component of the momentum crowd measure, and $Rank^{CoMOM_{pred_t}}$ is the rank of the $CoMOM_{resid}$ component of the momentum crowd measure. The ranks are based on quintile sorts for the corresponding variables. The $CoMOM_{pred}$ and $CoMOM_{resid}$ components are estimated based on regression (11) in the text. The sample period is from 1968 to 2022.

	Coefficient	Pr>ChiSq	
β_0	-0.4383	0.0154	
β_1	0.1124	0.0476	
β_2	-0.0922	0.1020	
	Odds Ratio		
	Estimate	Confidence Limits	
β_1	1.12	1.00	1.25
β_2	0.90	0.80	0.98

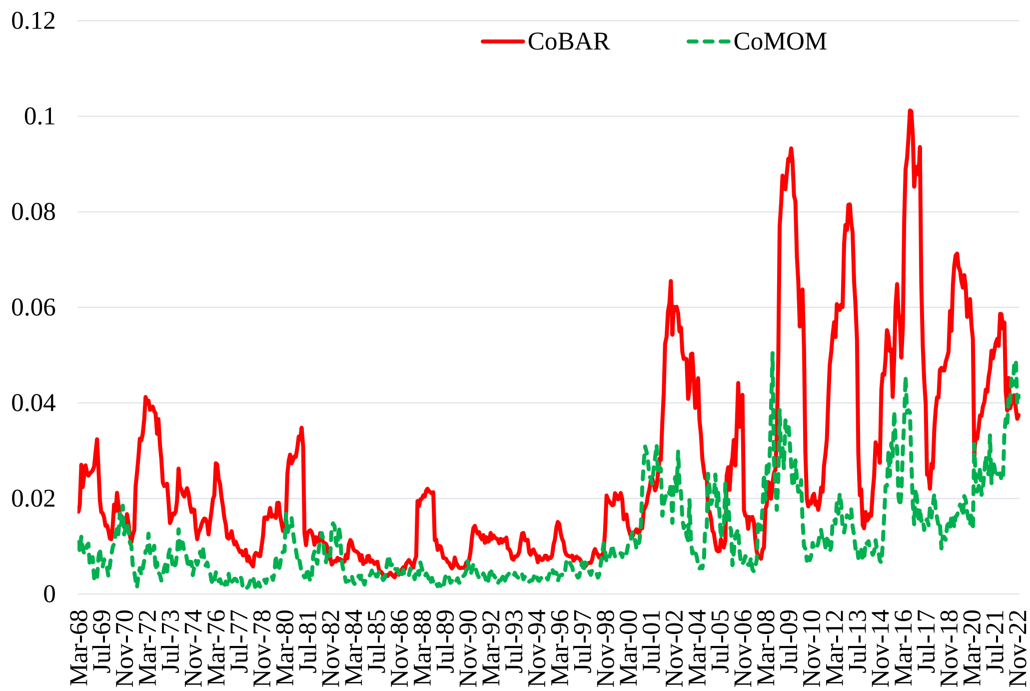

Figure 1. **Time series of Crowding Measures $CoBAR$ and $CoMOM$**
This figure plots the time series of the two crowding measures $CoBAR$ and $CoMOM$. $CoBAR$ (solid red line) is a measure of arbitrage activity in the betting-against-beta strategy and $CoMOM$ (dashed green line) is a measure of arbitrage activity in the momentum strategy. The measures are computed based on average pairwise correlations between residual returns of the extreme deciles of each strategy, as described in the main text. The sample period is from 1968 to 2022.

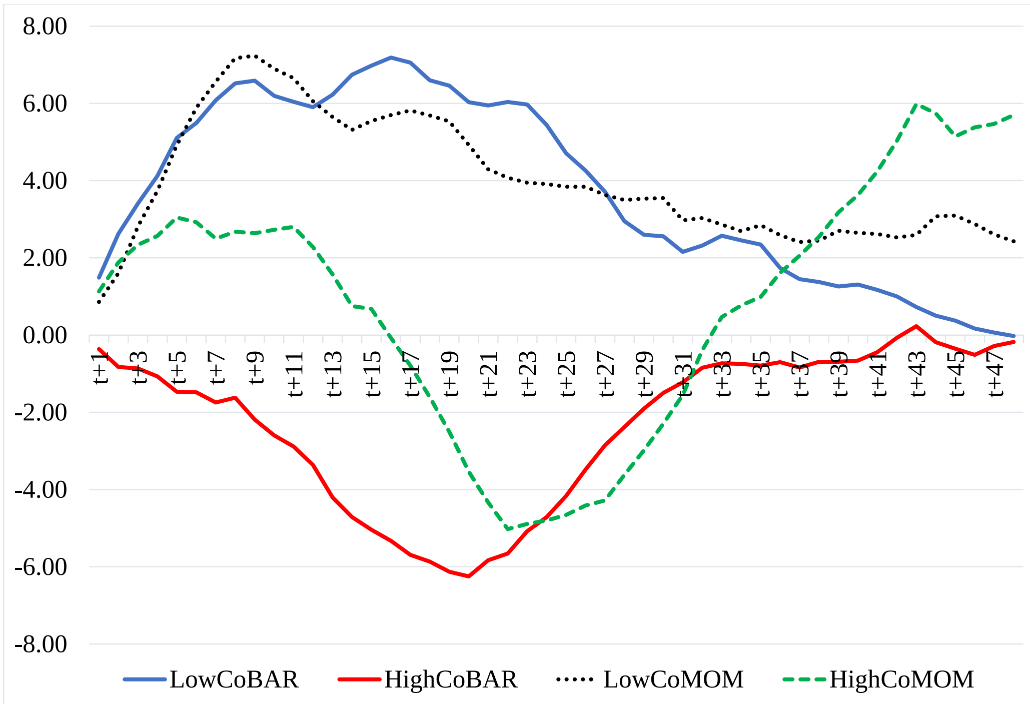

Figure 2. **Cumulative Returns to Momentum Conditional on Crowding Measures**
This figure plots the cumulative returns to the momentum strategy as a function of lagged crowding measures. At the end of each month, all stocks are sorted into deciles based on their lagged 11-month cumulative returns (skipping the most recent month). Stocks with prices below $5 and/or that are in the bottom NYSE size decile are excluded from the sample. All months are classifies into five groups based on $CoBAR$ and $CoMOM$. The figure plots the cumulative returns to the value-weighted momentum strategy for 48 months after portfolio formation, following low and high $CoBAR$ and $CoMOM$. The solid red line corresponds to cumulative momentum returns following High $CoBAR$, the solid blue line corresponds to cumulative momentum returns following Low $CoBAR$, the dashed green line corresponds to cumulative momentum returns following High $CoMOM$, and the dotted black line corresponds to cumulative momentum returns following Low $CoMOM$. The sample period is from 1968 to 2022.